THE BOLTZMANN EQUATION AND NEWTON'S METHOD

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A method much used in functional analysis for solving nonlinear equations is applied to the Boltzmann equation.

We investigate a possible mathematical descripton of a gas consisting of $N \gg 1$ identical classical particles of mass m located in a macroscopic volume V with the system being considered asymptotically large, i.e., $N \to \infty$, $V \to \infty$, and $N/V \equiv n = \text{const}$.

We use the following notation: $\mathbf{v}$, $\underset{*}{\mathbf{v}}$ are particle velocities before a collision leading to the appearance of the velocities $\mathbf{v}'$, $\underset{*}{\mathbf{v}}'$; $f \equiv f(t, \mathbf{r}, \mathbf{v})$, $\underset{*}{f} \equiv f(t, \mathbf{r}, \underset{*}{\mathbf{v}})$, $f' \equiv f(t, \mathbf{r}, \mathbf{v}')$, $\underset{*}{f}' \equiv f(t, \mathbf{r}, \underset{*}{\mathbf{v}}')$; $K \equiv K(|\mathbf{v}-\underset{*}{\mathbf{v}}|, \vartheta, \varphi)$ is the kernel of the collision integral containing the differential cross section $\sigma(|\mathbf{v}-\underset{*}{\mathbf{v}}|, \vartheta, \varphi)$ for particle scattering by the angles ϑ and φ in the center of mass system; $U(\mathbf{r})$ is the potential of the external field; $d\omega \equiv d\vartheta d\varphi$; $d\underset{*}{\mathbf{v}} \equiv d\underset{*}{v}_1 d\underset{*}{v}_2 d\underset{*}{v}_3$.

The equation for the function f can be written

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} - \frac{1}{m}\frac{\partial U(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{v}}\right) f = \int K(f'\underset{*}{f}' - f\underset{*}{f})\, d\omega d\underset{*}{\mathbf{v}},$$
$$f(0, \mathbf{r}, \mathbf{v}) = f_0(\mathbf{r}, \mathbf{v}) \equiv f_0. \tag{1}$$

We make certain assumptions with respect to the quantities appearing in it.

1. The particles interact through a field having a potential $\varphi(r_{ij})$ (r_{ij} is the distance between the i-th and j-th molecules) which is spherically symmetrical and which has a finite radius of action r_0, for example, for

$$\varphi(r_{ij}) = \begin{cases} \varphi_0 r_{ij}^{-(\nu-1)}, & r_{ij} \leqslant r_0 \\ 0, & r_{ij} > r_0 \end{cases} \qquad \nu > 3, \quad \varphi_0 = \text{const} > 0, \tag{2}$$

we obtain the kernel K in the form (see [1-3])

$$K \equiv |\mathbf{v} - \underset{*}{\mathbf{v}}|^{\gamma} k(\vartheta) \geqslant 0, \qquad \gamma \equiv \frac{\nu-5}{\nu-1}, \tag{3}$$

with

$$\int k(\vartheta)\, d\omega \equiv \tilde{k} < \infty. \tag{3a}$$

Consequently, the total particle interaction cross section S is finite. There are other well-known methods for introducing a finite S; apparently the selection of any particular one is of no great significance [4].

2. All functions appearing in Eq. (1) are continuous.

3. $U(\mathbf{r}) \geq 0$.

4. There is a constant $b > 0$ such that

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$$|f_0(\mathbf{r},\ \mathbf{v})\exp\{bE(\mathbf{r},\ \mathbf{v})\}| < \infty,\quad E(\mathbf{r},\ \mathbf{v}) \equiv \frac{m}{2}|\mathbf{v}|^2 + U(\mathbf{r}).$$

5. All collisions between particles are elastic; therefore:

$$|\mathbf{v}|^2 + |\underset{*}{\mathbf{v}}|^2 = |\mathbf{v}'|^2 + |\underset{*}{\mathbf{v}}'|^2,$$

$$\mathbf{v}' = \mathbf{v} + \mathbf{n}(\mathbf{n},\ \mathbf{v} - \underset{*}{\mathbf{v}}),$$

$$\underset{*}{\mathbf{v}}' = \underset{*}{\mathbf{v}} - \mathbf{n}(\mathbf{n},\ \mathbf{v} - \underset{*}{\mathbf{v}}),$$

$$\mathbf{n} \equiv \{\cos\vartheta,\ \sin\vartheta\cos\varphi,\ \sin\vartheta\sin\varphi\},$$

where the angle φ is arbitrary and ϑ is easily determined ([1, 3]) for a given potential $\varphi(\mathbf{r}_{ij})$.

The methods developed for the solution of Eq. (1), which are now rather numerous (see [1-3], for example), do not, as a rule, allow one to establish in what sense the approximate solution approaches the true solution (the actual existence of which is only assumed). The method developed below is free of such a deficiency.

Newton's Method. In functional analysis, the following is well known (see [5, 6]). Let X and Y be Banach space and B an operator mapping from X into Y. We assume that in the sphere $S(x_0,\ r) \subset X$, the operator B has a Frechet derivative B'_X which satisfies the Lipschitz condition

$$\|B'_{x_1} - B'_{x_2}\| \leqslant l\|x_1 - x_2\|_X,\quad l = \text{const}, \tag{6}$$

and the operator $(B'_{X_0})^{-1}$ exists. Then with respect to the equation

$$Bx = 0 \tag{7}$$

one can show the following.

THEOREM. If $\|(B'_{X_0})^{-1}\| \equiv \mu$, $\|(B'_{X_0})^{-1}Bx_0\|_X \equiv \varkappa$, and l is the constant appearing in inequality (6), then for

$$\mu\varkappa l < \frac{1}{4} \tag{8}$$

Eq. (7) has in the sphere $\|x - x_0\|_X \leq (1 - \sqrt{1 - 4\mu\varkappa l})/2\mu l$ a unique solution x and the sequence defined by the recurrence relation

$$x_{n+1} = x_n - (B'_{x_0})^{-1}Bx_n, \tag{9}$$

converges to it.

The modified Newton's method for the solution of Eq. (7) consists of its replacement by the solution for the series $B'_{X_0}x_{n+1} = B'_{X_0}x_n - Bx_n$.

Equation. We introduce a new unknown function $h(t,\ \mathbf{r},\ \mathbf{v})$ by means of the identity

$$f(t,\ \mathbf{r},\ \mathbf{v}) \equiv h(t,\ \mathbf{r},\ \mathbf{v})\exp\{-\beta E(\mathbf{r},\ \mathbf{v})(2t_1 - t)\}, \tag{10}$$

where $t \in (0,\ t_1)$, and β is a scalar parameter of dimensionality $[t]^{-1}[E]^{-1}$ related to t_1 by the equality $2\beta_1 = b$.

The function $h \equiv h(t,\ \mathbf{r},\ \mathbf{v})$ satisfies the equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} - \frac{1}{m}\frac{\partial U(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{v}} + \beta E(\mathbf{r},\ \mathbf{v})\right)h = \int K_e(h'\underset{*}{h}' - h\underset{*}{h})\,d\omega d\underset{*}{\mathbf{v}}, \tag{11}$$

$$\begin{gathered} h(0,\ \mathbf{r},\ \mathbf{v}) = h_0 \equiv f_0\exp\{bE(\mathbf{r},\ \mathbf{v})\}, \\ K_e \equiv K\exp\{-\beta E(\mathbf{r},\ \underset{*}{\mathbf{v}})(2t_1 - t)\}. \end{gathered} \tag{11a}$$

For the following, it is necessary to know some properties of the operator $[\ \ ]_s$, a backward displacement by a time s in accordance with the characteristics of the equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} - \frac{1}{m}\frac{\partial U(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial}{\partial \mathbf{v}}\right)u = 0$$

(see [7]). Its effect on any function $g(t,\ \mathbf{r},\ \mathbf{v})$ is that the first argument takes on the value $t - s$, and $\mathbf{r}$ and $\mathbf{v}$ take on those values for position and velocity which a particle of the gas under consideration, moving in the field $U(\mathbf{r})$ without collision, must have at the time $t - s$ in order that they respectively equal $\mathbf{r}$ and $\mathbf{v}$ at the time t. From this definition, we immediately conclude:

a) $[\gamma_1 x(t, \mathbf{r}, \mathbf{v}) + \gamma_2 y(t, \mathbf{r}, \mathbf{v})]_S = \gamma_1 [x(t, \mathbf{r}, \mathbf{v})]_S + \gamma_2 [y(t, \mathbf{r}, \mathbf{v})]_S$, $\gamma_1, \gamma_2 = \text{const}$;

b) $|[x(t, \mathbf{r}, \mathbf{v})]_S| = [|x(t, \mathbf{r}, \mathbf{v})|]_S$;

c) if the function $x(t, \mathbf{r}, \mathbf{v})$ is continuous, then the function $[x(t, \mathbf{r}, \mathbf{v})]_S$ is also continuous.

In addition, since $E(\mathbf{r}, \mathbf{v})$ is the total energy of a particle in the field $U(\mathbf{r})$, it is obvious that

$$[E(\mathbf{r}, \mathbf{v})]_s = E(\mathbf{r}, \mathbf{v}). \tag{12}$$

To the problem (11), (11a), there corresponds an integral equation which can be written, keeping Eq. (12) in mind,

$$h = \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\} \left[\int K_e (\overset{*}{h'}h' - \overset{*}{h}h)\, d\omega d\overset{*}{\mathbf{v}}\right]_{t-\tau} d\tau + [h_0]_t \exp\{-\beta E(\mathbf{r}, \mathbf{v}) t\}, \tag{13}$$

or, in operator form,

$$Bh = 0, \tag{14}$$

if by definition

$$B \equiv E + \{\ |\ \} + j, \tag{15}$$

$$\{x_1 | x_2\} \equiv \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\} \left[\int K_e (\overset{*}{x_1}x_2 - \overset{*}{x_1'}x_2')\, d\omega d\overset{*}{\mathbf{v}}\right]_{t-\tau} d\tau, \tag{15a}$$

$$j \equiv -[h_0]_t \exp\{-\beta E(\mathbf{r}, \mathbf{v}) t\}; \tag{15b}$$

E is the unit operator.

We apply Newton's method to Eq. (14).

Solution. We designate by C the Banach space of bounded continuous functions dependent on t, $\mathbf{r}$, and $\mathbf{v}$ $(t \in (0, t_1)$, $|\mathbf{r}| \leq \infty$, $|\mathbf{v}| \leq \infty)$ with the norm

$$\|x\|_C = \max_{t, \mathbf{r}, \mathbf{v}} |x|.$$

The operator B maps C into C. Indeed, for arbitrary $x \in C$ we have

$$|\{x|x\}| \leqslant 2\|x\|_C^2 l(t, \mathbf{r}, \mathbf{v}),$$
$$l(t, \mathbf{r}, \mathbf{v}) \equiv \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\}\left[\int K_e d\omega d\overset{*}{\mathbf{v}}\right]_{t-\tau} d\tau. \tag{16}$$

But it is easy to show that

$$l(t, \mathbf{r}, \mathbf{v}) \leqslant \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\}\left[\int K \exp\left\{-\frac{b}{2} E(\mathbf{r}, \overset{*}{\mathbf{v}})\right\} d\omega d\overset{*}{\mathbf{v}}\right]_{t-\tau} d\tau$$

$$\leqslant \tilde{k} \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\}\left[|\mathbf{v}|^\gamma \int \exp\left\{-\frac{mb}{4}|\overset{*}{\mathbf{v}}|^2\right\} d\overset{*}{\mathbf{v}} + \int |\overset{*}{\mathbf{v}}|^\gamma \exp\left\{-\frac{mb}{4}|\overset{*}{\mathbf{v}}|^2\right\} d\overset{*}{\mathbf{v}}\right]_{t-\tau} d\tau$$

$$= \tilde{k}\left(\frac{4\pi}{mb}\right)^{3/2} \int_0^t \exp\{-\beta E(\mathbf{r}, \mathbf{v})(t-\tau)\}\left[|\mathbf{v}|^\gamma + \frac{\Gamma(\gamma+3) D_{-(\gamma+3)}(0)}{\sqrt{2}(mb)^{\gamma/2}\Gamma\left(\frac{3}{2}\right)}\right]_{t-\tau} d\tau \tag{17}$$

(see [8]); here, $D_{-(\gamma+3)}$ is a parabolic cylinder function. Since $U(\mathbf{r}) \geq 0$, we have the relation

$$[|\mathbf{v}|^\gamma]_{t-\tau} \leqslant \left[\left(\frac{2}{m} E(\mathbf{r}, \mathbf{v})\right)^{\gamma/2}\right]_{t-\tau} = \left(\frac{2}{m} E(\mathbf{r}, \mathbf{v})\right)^{\gamma/2}.$$

Therefore

$$l(t, \mathbf{r}, \mathbf{v}) \leqslant \tilde{k}\left(\frac{4\pi}{mb}\right)^{3/2} \frac{1-\exp\left\{-\frac{b}{2} E(\mathbf{r}, \mathbf{v})\right\}}{\beta E(\mathbf{r}, \mathbf{v})}\left(\left(\frac{2}{m} E(\mathbf{r}, \mathbf{v})\right)^{\gamma/2} + \frac{\Gamma(\gamma+3) D_{-(\gamma+3)}(0)}{\sqrt{2}(mb)^{\gamma/2}\Gamma\left(\frac{3}{2}\right)}\right). \tag{18}$$

It then follows that $l(t, \mathbf{r}, \mathbf{v}) < \infty$, and further

$$l(t,\ \mathbf{r},\ \mathbf{v}) \underset{t_1 \to 0}{\to} 0, \tag{19}$$

since the quantity β increases without limit when $t_1 \to 0$. Therefore, since $j \in C$, the operator B acts in C. We shall show that it is Frechet differentiable, for which we consider the difference

$$B(x_0 + z) - Bx_0 = \{x_0 | z\} + \{z | x_0\} + z + \{z | z\}.$$

It is easy to show that

$$\frac{\|\{z | z\}\|_C}{\|z\|_C} \underset{\|z\|_C \to 0}{\longrightarrow} 0.$$

The operator

$$B'_{x_0} \equiv \{x_0 | \ \} + \{\ | x_0\} + E \tag{20}$$

is linear, operates in the space C and is bounded; it is therefore a strong derivative of the operator B at an (arbitrary) point $x_0 \in C$.

We verify that the operator B'_{x_0} always has an inverse, or, in other words, the following is valid.

THEOREM 1. The equation

$$B'_{x_0} z = s, \quad s \in C \tag{21}$$

has a unique solution for any $x_0 \in C$.

Indeed, if we look for the latter in the form

$$z(t,\ \mathbf{r},\ \mathbf{v}) \equiv \tilde{z}(t,\ \mathbf{r},\ \mathbf{v}) \exp\{\alpha t\}, \quad \alpha = \text{const} > 0, \tag{22}$$

then we obtain for the function $\tilde{z} \equiv \tilde{z}(t,\ \mathbf{r},\ \mathbf{v})$ the equation

$$\tilde{z} = \int_0^t \exp\{-(\beta E(\mathbf{r},\ \mathbf{v}) + \alpha)(t - \tau)\} \Big[\int K_e (x'_{0*}\tilde{z}' + x'_{0*}\tilde{z}' - x_0\tilde{z}_* - x_{0*}\tilde{z})\, d\omega d\mathbf{v}_* \Big]_{t-\tau} d\tau + s \exp\{-\alpha t\} \equiv L^\alpha \tilde{z}. \tag{23}$$

The Lipschitz constant for the operator L^α is

$$l_\alpha \equiv 4\|x_0\|_C \Big\| \int_0^t \exp\{-(\beta E(\mathbf{r},\ \mathbf{v}) + \alpha)(t - \tau)\} \Big[\int K_e d\omega d\mathbf{v}_* \Big]_{t-\tau} d\tau \Big\|_C$$

$$\leqslant 4\|x_0\|_C \tilde{k} \left(\frac{4\pi}{mb}\right)^{3/2} \left\| \frac{1 - \exp\{-(\beta E(\mathbf{r},\ \mathbf{v}) + \alpha) t_1\}}{\beta E(\mathbf{r},\ \mathbf{v}) + \alpha} \left(\left(\frac{2}{m} E(\mathbf{r},\ \mathbf{v})\right)^{\gamma/2} + \frac{\Gamma(\gamma + 3) D_{-(\gamma+3)}(0)}{\sqrt{2}(mb)^{\gamma/2}\Gamma\left(\frac{3}{2}\right)} \right) \right\|_C \tag{24}$$

and consequently has the property

$$l_\alpha \underset{\alpha \to \infty}{\to} 0. \tag{25}$$

For sufficiently large α, L^α is therefore a contraction operator. It transforms the sphere $S(0,\ r_\alpha)$, with a radius r_α satisfying the condition $\|s\|_C \le (1 - l_\alpha) r_\alpha$, into itself. But then in accordance with the principle of contraction reflections (see [5]), Eq. (23) has a unique solution in the sphere $S(0,\ r_\alpha)$ to which the sequence $\{z_{n+1} = L^\alpha z_n\}$, $z_0 \in S(0,\ r_\alpha)$ converges uniformly. Thereby, not only is Theorem 1 proved, but a method for solving equations like Eq. (23) is obtained.

It is now easy to show the following.

THEOREM 2. For any choice of the element x_0 and sufficiently small t_1, Eq. (14) has a solution which is unique in some sphere with center x_0.

In fact, in accordance with the properties of the norm of a linear operator

$$\|B'_{x_1} - B'_{x_2}\| = \sup_{\|s\|_C \leqslant 1} \|(B'_{x_1} - B'_{x_2}) s\|_C = \sup_{\|s\|_C \leqslant 1} \|B'_{x_1}s - B'_{x_2}s\|_C \leqslant 4\|l(t,\ \mathbf{r},\ \mathbf{v})\|_C \|x_1 - x_2\|_C = l\|x_1 - x_2\|_C,$$
$$l \equiv 4\|l(t,\ \mathbf{r},\ \mathbf{v})\|_C. \tag{26}$$

Using property (19) and the boundedness of the operator $(B'_{x_0})^{-1}$ (which follows from the boundedness of B'_{x_0}, see [9]), the quantity $\mu\varkappa l$ for arbitrary $x_0 \in C$ can be made so small that condition (8) is satisfied. Indeed, since the minimum radius of the sphere in which a solution of Eq. (21) exists is easily estimated, we obtain for $\|(B'_{x_0})^{-1}\|$ and $\|(B'_{x_0})^{-1} B_{x_0}\|_C$ the following inequalities:

$$\|(B'_{x_0})^{-1}\| \leqslant \frac{\exp\{\alpha t_1\}}{1-l_\alpha}, \tag{27}$$

$$\|(B'_{x_0})^{-1} Bx_0\|_C \leqslant \frac{\exp\{\alpha t_1\}}{1-l_\alpha} \|Bx_0\|_C. \tag{28}$$

The value of t_1 for which condition (8) is satisfied is determined from the knowledge of the upper bound (18) of the function $l(t, \mathbf{r}, \mathbf{v})$.

Thus for a sufficiently small interval $(0, t_1)$, the solution of Eq. (14) is the limit of the sequence of solutions for the array

$$B'_{x_0} x_{n+1} = B'_{x_0} x_n - Bx_n, \tag{29}$$

the equations of which are very much like Eq. (21) with Theorem 1 being valid for them also. The rate of convergence of this sequence is determined by means of the inequality (see [5])

$$\|x - x_n\|_C \leqslant \frac{q^n \varkappa}{1-q}, \qquad q \equiv \frac{1}{2}(1 - \sqrt{1 - 4\mu\varkappa l}). \tag{30}$$

Defining the function $h(t, \mathbf{r}, v)$ in an interval $(0, t_1)$ we assign $f(t_1, \mathbf{r}, v)$ as the initial condition for Eq. (1) in some time interval (t_1, t_2), etc. Obviously, the function f can be calculated in any time interval by this method under the restrictions introduced.

In order that the sequence (9) converge as rapidly as possible, the selection of the zeroth approximation x_0 should be made by using some physical concept or other or by taking the solution of a model equation as x_0.

The structure of Eq. (29) is similar to, and for Maxwellian particles with $U(\mathbf{r}) \equiv 0$ (where the function h is introduced by means of the identity $f \equiv h \exp\{-\chi|\mathbf{v}|^2\}$, $\chi = \text{const} > 0$) agrees with the integral form of the linearized kinetic equation. It is obvious that the solution of the latter is close to the solution of Eq. (1) when $h(0, \mathbf{r}, v)$ is so small that the sequence (9) converges in the interval $(0, t_1)$, $t_1 \sim \tau_\lambda$ (τ_λ is the mean free time of the particles) with all x_n starting with x_2 being neglected in comparison with x_1.

From the practical aspect, the Newton method (see [5]) is perhaps more useful where the role of sequence (9) is played by the following:

$$x_{n+1} = x_n - (B'_{x_n})^{-1} Bx_n. \tag{31}$$

It is characterized by more rapid convergence

$$\|x - x_n\|_C \leqslant \frac{1}{2^{n-1}} (2\mu\varkappa l)^{2^n - 1} \varkappa. \tag{32}$$

It is probable that for some special choice of x_0 the sequence (9) converges in the L_2 space of quadratically integrable functions. Indeed, at least for some $x_0 \in L_2$ the operator B'_{x_0} acts in L_2 (simplest example: if $x_0 \equiv 0$, then $B'_{x_0} \equiv E$ and $B'_{x_0} L_2 \subset L_2$). This makes it possible to attempt to solve Eq. (14) by Newton's method in L_2 space, which would make it possible to eliminate the requirement for continuity of the initial condition f_0.

The results presented permit simple generalization to the system

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \frac{\partial}{\partial \mathbf{r}} - \frac{1}{m_i} \frac{\partial U_i(\mathbf{r})}{\partial \mathbf{r}} \frac{\partial}{\partial \mathbf{v}_i}\right) f_i = \sum_{k=1}^{n} \int K_{ik} (f'_i f'_k - f_i f_k)\, d\omega d\mathbf{v}_\varkappa, \qquad i = 1, \ldots, n \tag{33}$$

and to more complex systems which, in particular, describe inelastic processes (see, for example, [10, 11]). The system (33) should obviously be considered as a vector equation with respect to the function

$$\mathbf{f} \equiv \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}, \qquad \mathbf{f} \in \prod_{i=1}^{n} C_i,$$

where by $\prod_{i=1}^{n} \underset{i}{C}$ is understood the product of spaces $\underset{i}{C}$ of bounded continuous functions dependent on t, $\mathbf{r}$, and $\mathbf{v}_i$. The corresponding theorems are nearly the same as those proven above.

NOTATION

V	is the volume of system under study;
N	is the number of particles in it,
N/V	is the density of particles;
t	is the time;
$\mathbf{r}$	is the vector-radius;
m	is the mass of particles;
v	is its velocity;
$f, \underset{*}{f}, f', \underset{*}{f'}$	are the distribution functions;
$U(\mathbf{r})$	is the external field potential;
σ	is the differential section of particle scattering by angles ϑ, φ;
S	is the total scattering section;
K	is the kernel of collision integral;
$\varphi(r_{ij})$	is the potential of molecular field;
r_{ij}	is the distance between i-th and j-th particles;
f_0	is the function of distribution at time moment t = 0;
φ_0, ν	are the constants in the law of potential variation $\varphi(r_{ij})$;
r_0	is the radius of action $\varphi(r_{ij})$;
$k(\vartheta)$	is the angular portion of kernel K;
γ	is the power exponent in relation K versus $\lvert v - v \rvert$;
X, Y, C	are the Banach spaces;
B	is the operator;
B'_x	is its derivative;
$b, l, l_\alpha, \alpha, \beta, \chi, \gamma_1, \gamma_2, \mu, \varkappa$	are the scalar constants;
h	is the new unknown function;
h_0	is the value at time moment t = 0;
$l(t, \mathbf{r}, \mathbf{v})$	is the auxiliary function;
$E(\mathbf{r}, \mathbf{v})$	is the total energy of particle;
$x, z, \tilde{z}, x_n, z_n$	are the elements of space C;
L^α	is the linear operator in it;
Γ	is the gamma-function;
$D_{-(\gamma+3)}$	is the function of parabolic cylinder;
$S(x_0, r)$, $S(0, r_\alpha)$	are the spheres in space C;
$[\;]_s$	is the shear operator;
$\lVert\;\rVert_C$	is the norm of element of space C.

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